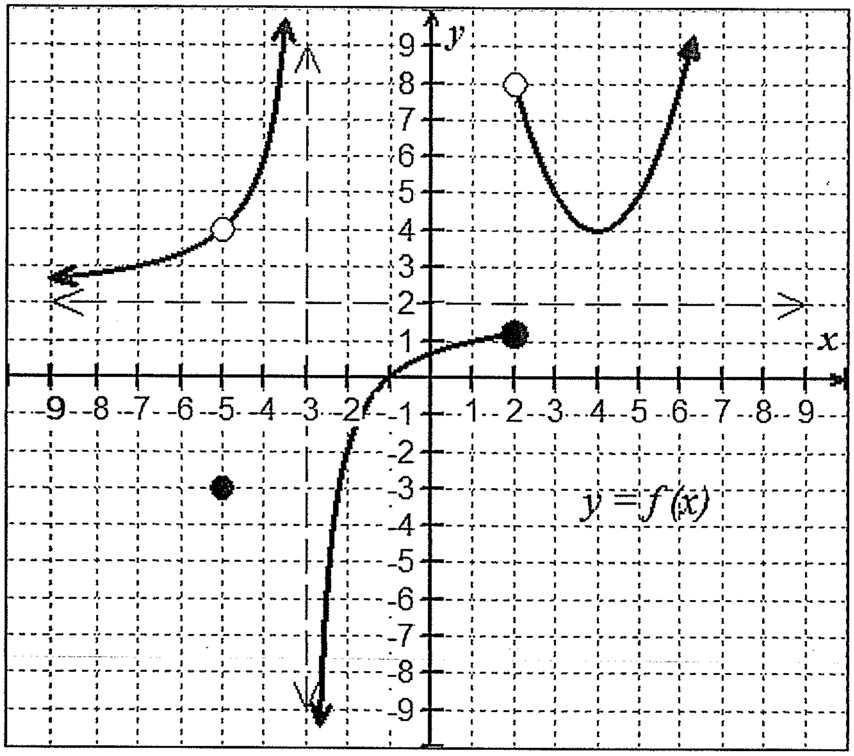
Understanding Limits Graphically and Numerically

Consider the graph of the function $f(x)$, graphed below:

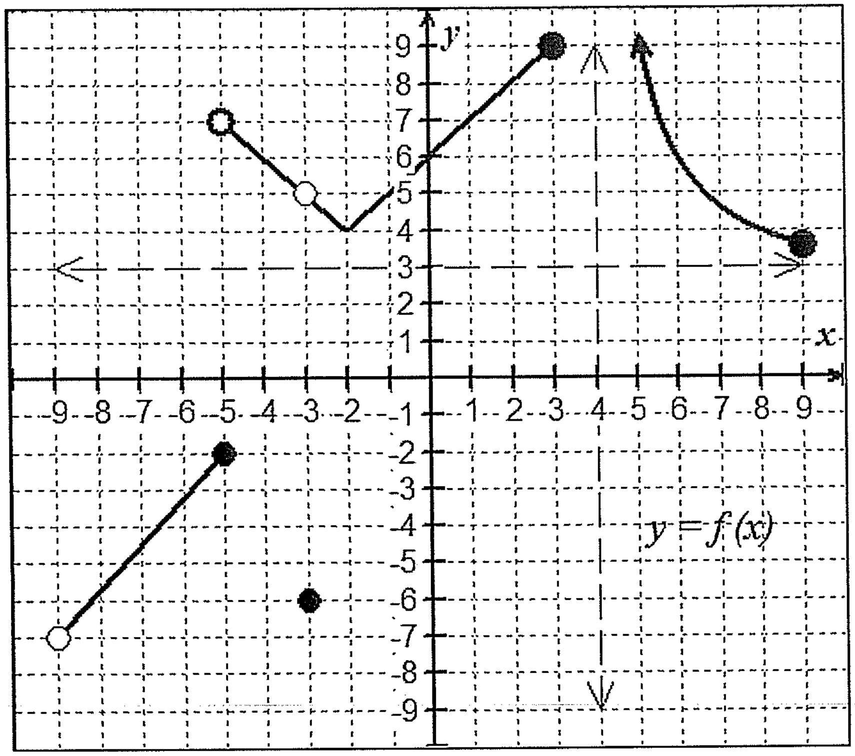


Using the graph, find the value of each of the following limits. If a limit does not exist, explain why.

A. $\lim_{x\to -3^-} f(x)$ ∞	B. $\lim_{x\to -5} f(x)$ 4	C. $\lim_{x\to 4} f(x)$ 4
D.) $\lim_{x\to 2^+} f(x)$ 8	E.) $\lim_{x\to 2^-} f(x)$ 1	F.) $\lim_{x\to 2} f(x)$ DNE right hand does not equal left
G.) $\lim_{x\to -1} f(x)$ 0	H.) $\lim_{x\to -\infty} f(x)$ 2	I.) $\lim_{x\to \infty} f(x)$ ∞

 Jean Adams

Now you give it a try. Consider the graph shown below to find the value of each of the following limits. If a limit does not exist, explain why.



A. $\lim_{x \to -5^+} f(x)$ 7	B. $\lim_{x \to -2} f(x)$ 4	C. $\lim_{x \to -3} f(x)$ 5
D.) $\lim_{x \to 3^+} f(x)$ DNE no function as $X \to 3^+$	E.) $\lim_{x \to 3^-} f(x)$ 9	F.) $\lim_{x \to -5^-} f(x)$ -2
G.) $\lim_{x \to 0} f(x)$ 6	H.) $\lim_{x \to -9} f(x)$ DNE no function as $X \to 9^-$	I.) $\lim_{x \to 4^+} f(x)$ ∞

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Limits are the "backbone" of understanding that connect algebra and geometry to the mathematics of calculus. In basic terms, a limit is just a statement that tells you what height a function *INTENDS TO REACH* as you get close to a specific x-value. Recall from Pre-Calculus that you evaluated three types of limits. Complete the table below:

PROPER LIMIT NOTATIONS		
TYPE OF LIMIT	PROPER NOTATION	VERBALLY:
Right-hand limit	$\lim_{x \to c^+} f(x)$	limit as x approaches c from the right
Left-hand limit	$\lim_{x \to c^-} f(x)$	limit as x approaches c from the left
General limit	$\lim_{x \to c} f(x)$	limit as x approaches c

Consider the function shown below.
Say you want to find $\lim_{x \to 4^+} f(x)$, the positive sign in the limit notation indicates a right-hand limit. If you think of the function as a highway and imagine you are traveling along the graph of $f(x)$ toward $x = 4$ FROM THE RIGHT, NOT TO THE RIGHT, and you stop at the vertical line $x = 4$, the y-value where you stop is 3. Therefore, $\lim_{x \to 4^+} f(x) = 3$.

You will use this graph to explore the limits for the problems on the next page.

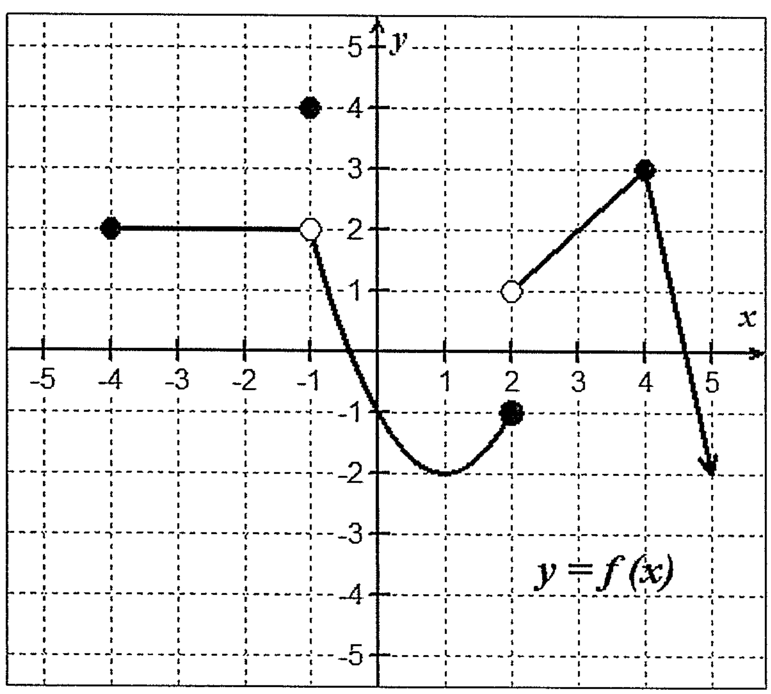


Figure 1-1

EX #1: Use Figure 1-1 to find the function values and evaluate each of the following limits:

1. $f(2)$ -1	2. $f(-1)$ 4
3. $\lim_{x\to 4^-} f(x)$ 3	4. $\lim_{x\to 2^+} f(x)$ 1
5. $\lim_{x\to 2^-} f(x)$ -1	6. $\lim_{x\to -1^+} f(x)$ 2
7. $\lim_{x\to -1^-} f(x)$ 2	8. $\lim_{x\to -4^+} f(x)$ 2
9. $\lim_{x\to -4^-} f(x)$ dne	10. $\lim_{x\to -1} f(x)$ 2
11. $\lim_{x\to 2} f(x)$ dne	12. $\lim_{x\to 5} f(x)$ -2
13. $\lim_{x\to 0} f(x)$ -1	14. $\lim_{x\to 1} f(x)$ -2

EX #2: Think about this!

If we think of the function as a highway, then the point at $(2,-1)$ could be considered the end of the road, while the point at $(-1,2)$ is more like a "pothole." How would you describe the points located at

$(2,1)$: dead end with no barrier

$(4,3)$: bump in road

Hopefully, this analogy gives you a visual reference for understanding limits from a graphical approach. Let's get a little more formal with our definition now.

When finding limits, ask yourself, "What is happening to y as x gets close to a certain number?" You are finding the ***y*-value** for which the function is approaching as x approaches c.

LIMIT EXISTENCE THEOREM:

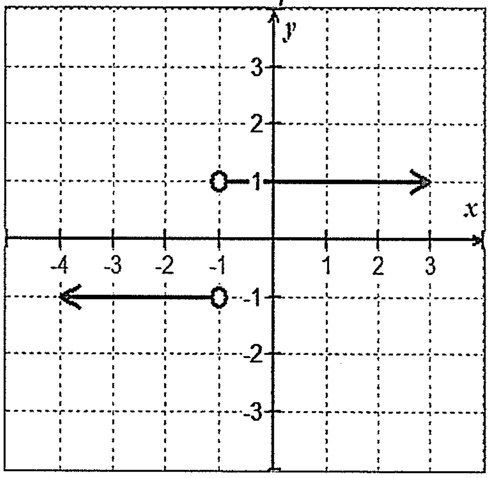
$\lim_{x\to c} f(x)$ exists if and only if

$\lim_{x\to c^-} f(x) = \lim_{x\to c^+} = L$ where L is a real #

Verbally: The limit as x approaches c on $f(x)$ will exist if and only if the limit as x approaches c from the left is equal to the limit as x approaches c from the right.

EX #3: Limits can fail to exist in three situations:

CASE 1: Jump



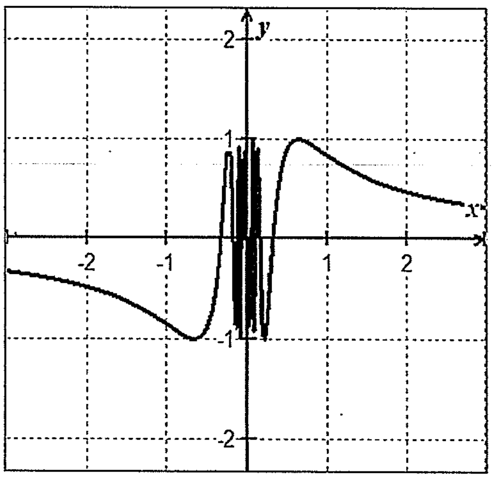
Justify why the limit does not exist at $x = -1$ for $f(x) = \frac{|x+1|}{x+1}$

$\lim_{x \to -1^+} f(x) = 1$

$\lim_{x \to 1^-} f(x) = -1$

$\lim_{x \to -1^+} f(x) \neq \lim_{x \to -1^-} f(x)$

CASE 2: Oscillation



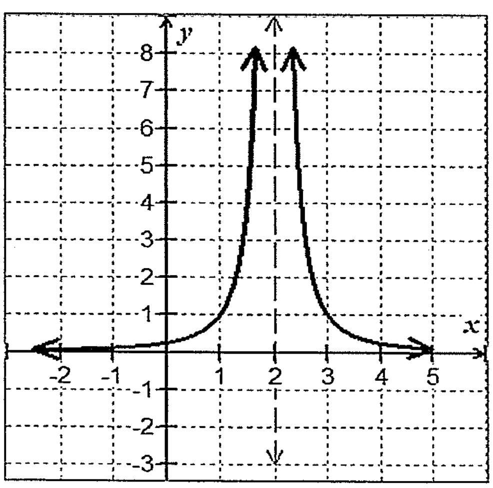
Justify why the limit does not exist at $x = 0$ for $f(x) = \sin\left(\frac{1}{x}\right)$

At $x = 0.1$ $\lim_{x \to 0^+} f(x) < 0$

at $x = -0.1$ $\lim_{x \to 0^-} f(x) > 0$

limit DNE

Case 3: Unbounded behavior



Justify why the limit does not exist at $x = 2$ for $f(x) = \frac{1}{(x-2)^2}$

$\lim_{x \to 2^+} f(x) = \infty$

$\lim_{x \to 2^-} f(x) = \infty$

$\lim_{x \to 2} f(x) = \infty$

Limit does not exist since ∞ is not a number.

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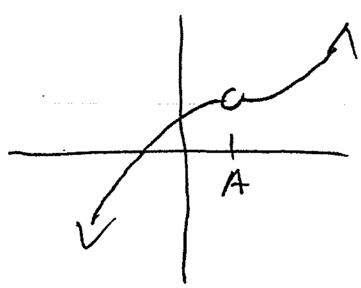
EX #4: YOU OWN IT! In the box below, complete the sentence in your own words.

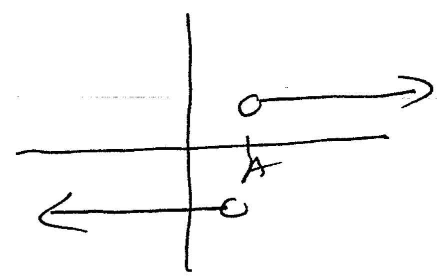
In order for the GENERAL LIMIT to exist, the function:

EX #5: Sketch a graph to satisfy each set of conditions.

1. $f(a)$ is undefined
2. $x = a$ is a point discontinuity
3. $\lim_{x \to a} f(x)$ exists



1. $\lim_{x \to a} f(x)$ DNE
2. ~~$x = a$ is a jump discontinuity~~
3. $f(a)$ is undefined



EX #6: Finding limits from a table of values

Now, consider the function $f(x) = \frac{x-3}{x^2+2x-15}$. Complete the table below to find the limit as $x \to 3$.

x	2.9	2.99	2.999	3	3.001	3.01	3.1
$f(x)$	0.1265	0.1251	0.125	DNE	0.1249	0.1248	0.1234

Based on your analysis, what are the values of each of the limits below?

$\lim_{x \to 3^-} f(x) = 0.125$	$\lim_{x \to 3^+} f(x) = 0.125$	$\lim_{x \to 3} f(x) = 0.125$