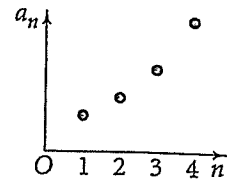
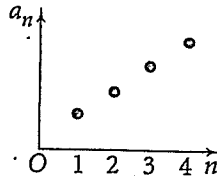




Need Help?

To review exponential curves, go to Lesson 8-2.

The graphs of arithmetic and geometric sequences have different shapes.



* Arithmetic graphs are linear.

Geometric graphs are exponential.

You can find the **geometric mean** of any two positive numbers by taking the positive square root of the product of the two numbers.

$$\text{geometric mean} = \sqrt{\text{product of the two numbers}}$$

You can use the geometric mean to find a missing term of a geometric sequence.

3

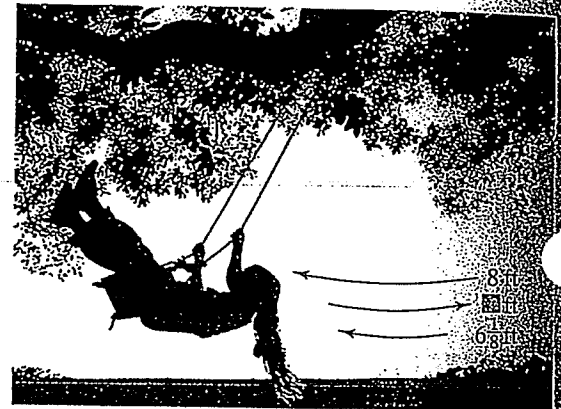
EXAMPLE

Real-World Connection

Physics When a child swings without being pushed, air resistance causes the length of the arc of the swing to decrease geometrically. Find the missing arc length.

Find the geometric mean of the two arc lengths.

$$\begin{aligned} \text{geometric mean} &= \sqrt{8 \cdot 6\frac{1}{8}} \\ &= \sqrt{49} \\ &= 7 \end{aligned}$$



On the second swing, the length of the arc is 7 ft.

Check Understanding

Find the missing term of each geometric sequence.

a. 20, \square , 80, ...

b. 3, \square , 18.75, ...

c. 28, \square , 5103, ...

EXERCISES

For more practice, see *Extra Practice*.

Practice and Problem Solving

A Practice by Example

Example 1
(page 601)

Is the given sequence geometric? If so, identify the common ratio and find the next two terms.

1. 1, 2, 4, 8, ...

2. 1, 2, 3, 4, ...

3. 1, -2, 4, -8, ...

4. -1, 1, -1, 1, ...

5. 10, 4, 1.6, 0.64, ...

6. 7, 0.7, 0.07, 0.007, ...

7. 18, -6, 2, $-\frac{2}{3}$, ...

8. $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \dots$

9. 10, 15, 22.5, 33.75, ...

10. 2, -10, 50, -250, ...

11. -1, -6, -36, -216, ...

12. $\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \dots$

Example 2
(page 601)

Write the explicit formula for each sequence. Then generate the first five terms.

13. $a_1 = 5, r = -3$ 14. $a_1 = 0.0237, r = 10$ 15. $a_1 = \frac{1}{2}, r = \frac{2}{3}$
 16. $a_1 = 1, r = 0.5$ 17. $a_1 = 100, r = -20$ 18. $a_1 = 7, r = 1$
 19. $a_1 = 1024, r = 0.5$ 20. $a_1 = 4, r = 0.1$ 21. $a_1 = 10, r = -1$

Example 3
(page 602)

Find the missing term of each geometric sequence.

22. 5, \square , 911.25, ... 23. 9180, \square , 255, ... 24. $\frac{2}{3}, \square, \frac{8}{45}, \dots$
 25. 3, \square , 0.75, ... 26. 5, \square , 2.8125, ... 27. 12, \square , 3, ...

B Apply Your Skills

Identify each sequence as *arithmetic*, *geometric*, or *neither*. Then find the next two terms.

28. 45, 90, 180, 360, ... 29. 25, 50, 75, 100, ... 30. 3, -3, 3, -3, ...
 31. 30, 35, 40, 45, ... 32. -5, 10, -20, 40, ... 33. 2, 1, 0.5, 0.25, ...
 34. 5, 6, 8, 11, 15, ... 35. 2, 2, 2, 2, ... 36. 1, 4, 9, 16, ...

Find the missing terms of each geometric sequence. (*Hint: The geometric mean of the first and fifth terms is the third term.*)

37. 19,683; \square ; \square ; \square ; 243; ... 38. 2.5, \square , \square , \square , 202.5, ...
 39. 12.5, \square , \square , \square , 5.12, ... 40. -4, \square , \square , \square , $-30\frac{3}{8}$, ...



Reading Math

For help with reading and solving Exercise 37, see p. 606.

41. **a. Open-Ended** Choose two positive numbers. Find their geometric mean.
b. Find the common ratio for a geometric sequence that includes the terms from part (a) in order from least to greatest or from greatest to least.
c. Find the 9th term of the geometric sequence from part (b).
d. Find the geometric mean of the term from part (c) and the first term of your sequence. What term of the sequence have you just found?

For the geometric sequence 3, 12, 48, 192, ..., find the indicated term.

42. 5th term 43. 7th term 44. 10th term
 45. 14th term 46. 17th term 47. n th term

Find the 10th term of each sequence.

48. $a_9 = 8, r = \frac{1}{2}$ 49. $a_{11} = 8, r = \frac{1}{2}$
 50. $a_9 = -5, r = -\frac{1}{2}$ 51. $a_{11} = -5, r = -\frac{1}{2}$
 52. $a_9 = -\frac{1}{3}, r = \frac{1}{2}$ 53. $a_{11} = -\frac{1}{3}, r = \frac{1}{2}$



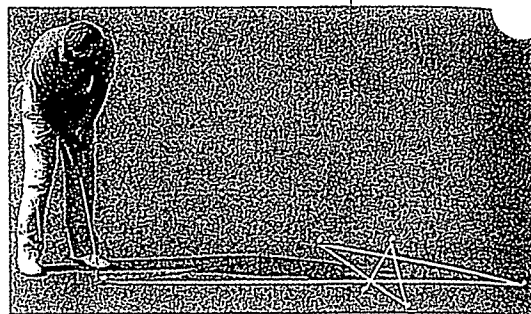
54. **Writing** Describe the similarities and differences between a common difference and a common ratio.

C Challenge

55. **Banking** Copy and complete the table below. Use the geometric mean. Assume compound interest is earned and no withdrawals are made.

Period 1	\$140.00	\$600.00	\$25.00	\$57.50	\$100.00	\$250.00
Period 2	\square	\square	\square	\square	\square	\square
Period 3	\$145.64	\$627.49	\$32.76	\$60.37	\$111.98	\$276.55

56. **Golf** Each of the putts misses the hole and continues past it for half the distance.
- Write a sequence to represent the ball's distance from the hole before each of his first six putts.
 - Is this sequence geometric? Explain your reasoning.
 - Write a recursive formula for the sequence.

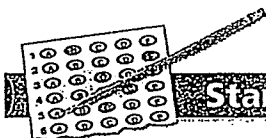


57. **Critical Thinking** How are the formulas for a geometric sequence similar to the formulas for an arithmetic sequence?
58. Suppose a balloon loses one fourth of its helium each day. The balloon starts with a volume of 5000 cm^3 .
- Write the geometric sequence that shows the amount of helium in the balloon at the start of each day for five days.
 - What is the common ratio of the sequence?
 - How much helium will be left in the balloon at the start of the tenth day?
 - Graph the sequence. Then sketch the graph.
 - Critical Thinking** How does the common ratio affect the shape of the graph?

Find a_1 for a geometric sequence with the given terms.

59. $a_5 = 112$ and $a_7 = 448$

60. $a_9 = \frac{1}{2}$ and $a_{12} = \frac{1}{16}$



Standardized Test Prep

Multiple Choice

61. Which geometric sequence DOES NOT include the term 100?
- 5, 10, 20, ...
 - 337.5, 225, 150, ...
 - $a_1 = 25, a_n = 2a_{n-1}$
 - $a_n = 4 \cdot 5^n$
62. What is the product of the geometric mean of 2 and 32 and the geometric mean of 1 and 4?
- 16
 - 19
 - 32
 - 256



Take It to the NET

Online lesson quiz at
www.PHSchool.com

Web Code: aga-1103

Quantitative Comparison

- Compare the boxed quantity in Column A with the boxed quantity in Column B. Choose the best answer.
- The quantity in Column A is greater.
 - The quantity in Column B is greater.
 - The two quantities are equal.
 - The relationship cannot be determined from the information given.

Each group of three terms represents a geometric sequence.

Column A

Column B

63. the missing term in 5, , 125

the missing term in 5, 10,

64. the missing term in 10, 100,

the missing term in , 200, 40

65. the missing term in 15, 45,

the missing term in , 280, 560

