Example 2

Problem Ricardo has 3 fewer apples than Jeremy. Let j stand for the number of apples that Jeremy has. Write an expression for the number of apples that Ricardo has.

Solution Think about the steps you take to find how many apples Ricardo has.

Suppose Jeremy has 10 apples. Ricardo has 3 fewer apples than Jeremy, or $10 - 3$ equals 7 apples.

Suppose Jeremy has 15 apples. Ricardo has 3 fewer apples than Jeremy, and $15 - 3$ equals 12 apples.

Use j for the number of apples that Jeremy has. Suppose Jeremy has j apples. Ricardo has 3 fewer apples than Jeremy. The expression $j - 3$ equals the number of apples that Ricardo has.

Habits of Mind

Establish a process
Keeping track of you steps is a very useful habit in algebra.

For Discussion

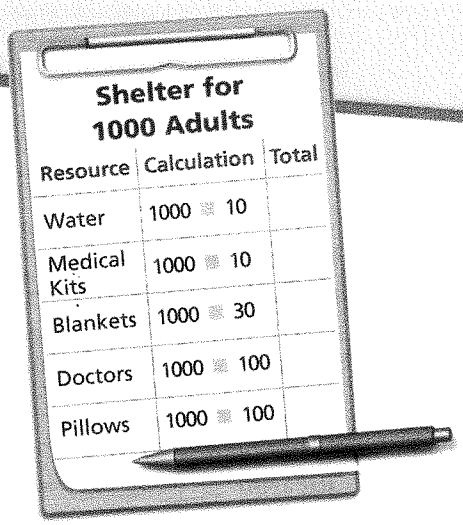
4. Hideki says, "I chose a number. I multiplied it by 7. Then I subtracted 4." Let h stand for Hideki's starting number. Write an expression for Hideki's ending number.

Exercises *Practicing Habits of Mind*

Check Your Understanding

1. Mary was born one year before Barbara. No matter how old they are, Mary will always be one year older than Barbara. Find the missing ages in years.

	Mary's Age	Barbara's Age
a.	11	■
b.	7	■
c.	53	■
d.	65	■
e.	m	■
f.	■	n



2. A relief group can use expressions to plan for disasters.
At a relief camp, you need many kinds of supplies.

- For each adult, the camp needs 10 gallons of water per week.
- For every 10 adults, the camp needs one medical kit.
- The camp needs 30 more blankets than the number of adults.
- For every 100 adults, the camp needs one doctor.
- The camp needs 100 more pillows than the number of adults.

Let a equal the number of adults in the camp.
Match each expression to one of the five items above.
Explain your answers.

a. $\frac{a}{100}$ **b.** $10 \cdot a$ **c.** $\frac{a}{10}$

d. $a + 100$ **e.** $a + 30$

3. Match each expression below to a set of steps I–V.

a. $2 \cdot x - 2$ **b.** $3 \cdot (x + 5)$ **c.** $2 \cdot (x - 2)$

d. $5 \cdot (x + 3)$ **e.** $3x + 5$

I. Choose any number.
Subtract 2.
Multiply it by 2.

II. Choose any number.
Multiply it by 2.
Subtract 2.

III. Choose any number.
Add 3.
Multiply it by 5.

IV. Choose any number.
Add 5.
Multiply it by 3.

V. Choose any number.
Multiply it by 3.
Add 5.

4. For each expression, write a number trick.

a. $2 \cdot x + 1$ **b.** $-2 \cdot (x - 1)$

c. $5 \cdot (x + 2) - 2$ **d.** $7 \cdot (3 \cdot (x + 1) - 2) - 9$

5. Use the number x as your starting number. Write the expression resulting from following the steps.

- Choose a number.
- Add 5.
- Subtract 11.
- Multiply by 2.
- Add 3.

On Your Own

6. Jeremy has 3 more apples than Ricardo. Find the number of apples Jeremy has.

	Number of Apples	
	Ricardo	**Jeremy**
a.	2	
b.	13	
c.	107	
d.	r	

7. Match each expression below to a set of steps I–IV.

a. $-7 \cdot x + 5$

b. $5 \cdot (x - 7)$

c. $7 \cdot (x - 5)$

d. $7 \cdot x - 5$

I. Choose any number.
Subtract 5.
Multiply by 7.

II. Choose any number.
Multiply by -7.
Add 5.

III. Choose any number.
Subtract 7.
Multiply by 5.

IV. Choose any number.
Multiply by 7.
Subtract 5.

8. **What's Wrong Here?** Travis writes an expression using the following steps.

- Start with x.
- Subtract 3.
- Multiply by 10.
- Subtract 13.

He writes the final expression $10x - 3 - 13$. Explain what he did wrong. Then find the correct expression.

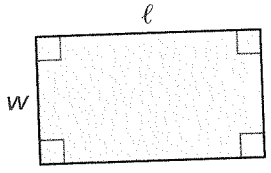
9. **Standardized Test Prep** David counts the quarters in his change jar. He has 24 more quarters than his brother Rob. If r is the number of quarters Rob has, which expression represents the number of quarters that David and Rob have in all?

A. $r + 24$ **B.** $2r - 24$ **C.** $2r$ **D.** none of these

10. A rectangle has length ℓ and width w. For a rectangle, write an expression for each of the following.

a. area

b. perimeter



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11. You overhear Mrs. Antonellis say, "During fifth period, I teach math. There are four other classes that meet during that period: computer, gym, history, and art. I notice some interesting things about the class sizes.

- For every two students in my math class, there is one student in computer class.
- For each student in math class, there are two students in gym class.
- There are two more students in my math class than in the history class.
- There are two more students in the art class than in my math class."

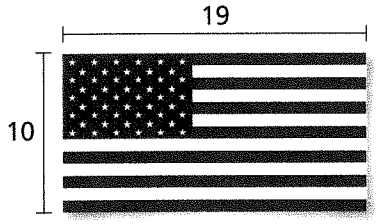
Let x equal the number of students in Mrs. Antonellis' math class. Using x, write an expression for the number of students in each class.

a. computer **b.** gym **c.** history **d.** art

e. Write an expression for the total number of students in the math, computer, gym, and history classes.

Maintain Your Skills

12. While in office, President Dwight D. Eisenhower standardized the proportions of United States flags. For every ten units of width, the flag must have nineteen units of length.

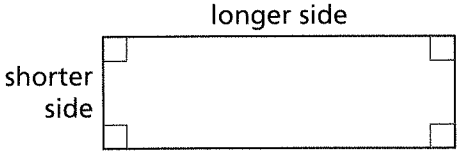


Find a flag's length, in inches, for each given width.

a. 20 inches **b.** 5 inches

c. 1 inch **d.** x inches

13. A computer company wants to make computers with rectangular cases. The length of the longer side is four times the length of the shorter side.



Find the length of the longer side, in centimeters, for each given length of the shorter side.

a. 20 cm **b.** 25 cm **c.** x cm

Check Your Understanding

1. A disaster relief group must provide beds, food, and water for a camp. It uses the following expressions for the quantities it needs. The expressions depend on two variables, the number of adults and the number of children. Let a = the number of adults. Let c = the number of children.
   - $a + c + 10$ beds
   - $5a + 2c$ pounds of food
   - $9a + 5c$ gallons of water

   How many beds and how much food and water are needed for 500 adults and 100 children? For 1000 adults and 300 children?

2. Evaluate $\frac{4x + 3x + 2x + x}{x}$ for each value of x.

   a. 3 **b.** 17 **c.** -2 **d.** 11

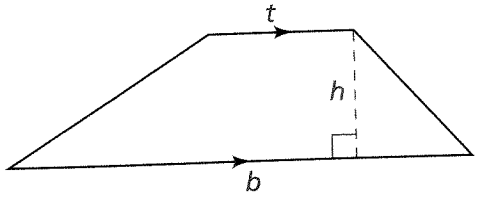
3. Maya says, "Choose a number. Subtract 1. Multiply by 3. Add 5." For each starting number given, what is your ending number?

   a. n **b.** 11 **c.** 1

4. Spiro says, "Choose a number. Multiply by 3. Subtract 2. Multiply by 5." For each starting number given, what is your ending number?

   a. n **b.** 4 **c.** $\frac{4}{3}$

5. A trapezoid is a shape with four sides that has one pair of parallel sides.



   You can find the area of a trapezoid using the three dimensions of the trapezoid labeled b, t, and h in the diagram. The expression for the area of a trapezoid is $\frac{(b + t)}{2} \cdot h$.

   Find the area of each trapezoid with the given dimensions.

   a. $b = 5, t = 3, h = 6$

   b. $b = 2, t = 3, h = 7$

   c. $b = \frac{3}{2}, t = \frac{1}{2}, h = \frac{7}{4}$

   d. $b = 8, t = 6, h = \frac{1}{4}$

The variables b an represent the leng of the bottom an top parallel sides, respectively. The h represents the h of the trapezoid.

6. Insert parentheses to make each equation true.

Sample $2 + 3 \cdot 7 \stackrel{?}{=} 35$

Solution $(2 + 3) \cdot 7 = 35$

a. $3 \cdot 7 + 3 \stackrel{?}{=} 30$

b. $-3 + 3 \cdot 5 + 11 \stackrel{?}{=} 11$

c. $-3 + 3 \cdot 5 + 11 \stackrel{?}{=} 45$

d. $25 - 5 + 4 \cdot 5 \stackrel{?}{=} 0$

e. $25 - 5 + 4 \cdot 5 \stackrel{?}{=} -20$

On Your Own

7. Boyle's Law in chemistry states that when temperature is constant, the pressure of a gas is inversely proportional to its volume. Let p represent the pressure, and let V represent the volume of the gas. At a certain temperature, $V = \frac{\text{constant}}{p}$. By what factor does the volume of the gas change if the pressure changes by each given factor?

a. 2 **b.** 3

c. 4 **d.** 5

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8. **What's Wrong Here?** Linda evaluates the expression $7 + 5x$ for $x = 3$ and $x = \frac{4}{5}$. When $x = 3$, she gets $7 + 5x = 7 + 5 \cdot (3) = 12 \cdot (3) = 36$.

When $x = \frac{4}{5}$, she gets $7 + 5x = 7 + 5 \cdot \left(\frac{4}{5}\right) = 12 \cdot \left(\frac{4}{5}\right) = \frac{48}{5}$.

What does Linda do wrong?

9. Evaluate the expression $5 + 3x$ for each value of x.

a. 3 **b.** -1

c. -5 **d.** 0

e. -11 **f.** -3

10. Evaluate the expression $(z + 3)^2 - 4$ for each value of z.

a. 3 **b.** -1

c. -5 **d.** 0

e. -11 **f.** -3

When you squeeze a balloon in one place, the increased pressure will force a bulge in another.

11. You can find a person's speed by dividing how far the person travels by the time spent traveling. The equation $r = \frac{d}{t}$ shows this relationship, where r is the rate (or speed), d is the distance, and t is the time.

a. Anh jogs 100 meters in 40 seconds. What is his rate in meters per second?

b. Isabel runs 50 yards in 25 seconds. What is her rate in yards per second?

c. Rosario jogs at a rate of 6 miles per hour. How far does she run in 30 minutes?

12. **Standardized Test Prep** Evaluate $14 - 6 + 32 \div 8 \cdot 2$.

A. 10 **B.** 6

C. 16 **D.** 12

Maintain Your Skills

13. Evaluate the expression $(x - 1) \cdot (x - 2) \cdot (x - 3) \cdot (x - 4) \cdot (x - 5)$ for each x value.

a. 1

b. 2

c. 3

d. 4

e. 5

f. What is similar about these five cases? Is there a sixth similar case? Explain.

14. Here are a complicated expression and a simple expression.

- $3x + (x - 1) \cdot (x - 2) \cdot (x - 3) \cdot (x - 4) \cdot (x - 5)$
- $3x$

Evaluate both expressions above for each x value.

a. 1

b. 2

c. 4

d. What are the x values that produce the same result in both expressions?

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2.4 Simplifying Expressions

You can take an expression that looks complicated and make it simpler to use.

Minds in Action episode 4

Tony I think I know your secret, Maya. The key is to use variables.

Maya Show me what you mean.

Tony Well, look at this trick. Choose a number. Add 3. Multiply by 2. Subtract 6.

The ending number will always be twice as large as the starting number.

Maya How do you know that?

Tony Watch. If I start with x, I can follow these steps.

Choose a number.	x
Add 3.	$x + 3$
Multiply by 2.	$2 \cdot (x + 3)$
Subtract 6.	$2(x + 3) - 6$

Now watch this. You can simplify $2(x + 3)$. Use the Distributive Property and multiply: $2 \cdot (x + 3) = 2 \cdot x + 2 \cdot 3 = 2x + 6$.

Since $2(x + 3) = 2x + 6$, $2(x + 3) - 6 = 2x + 6 - 6 = 2x$. So the ending number is $2x$.

Maya Very good, Tony. Now can you figure out my other tricks?

For Discussion

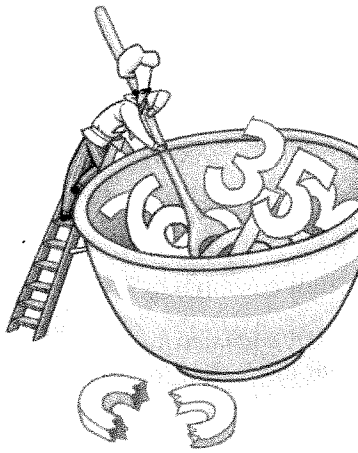
1. Here is a number trick.
   - Choose any number.
   - Multiply by 2.
   - Add 7.
   - Multiply by 5.
   - Add 25.
   - Divide by 10.
   - Subtract 6.

   What is the trick? Use expressions to show that it always works.

Check Your Understanding

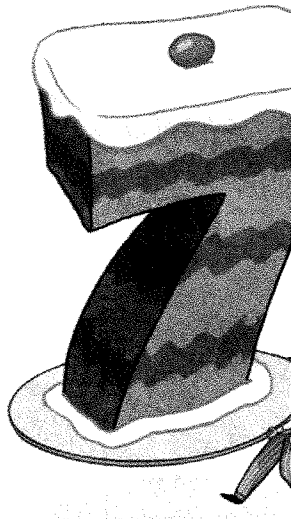
1. Here is a number trick similar to the trick in For Discussion Problem 1.
   - Choose any number.
   - Multiply by 2.
   - Add 7.
   - Multiply by 5.
   - Add 25.
   - Divide by 10.
   - Subtract your starting number.

   What is the trick? Will it work for any number? Explain by using expressions.



2. Here is a number trick with one missing step.
   - Choose any number.
   - Multiply by 2.
   - Add 7.
   - Multiply by 5.
   - ?
   - Divide by 10.
   - Subtract your starting number.
   - Your ending number is 7.

   What is the missing step?



3. Here is a number trick with the last step missing.
   - Choose any number.
   - Multiply by 3.
   - Add 5.
   - Multiply by 4.
   - Add 16.
   - Divide by 12.
   - ?

   For each result given below, what is the last step in the number trick?

   a. The ending number is the same as the starting number.

   b. The ending number is 3.

4. Evaluate the expression $\frac{x + 2x + 3x + 4x + 5x}{x}$ for each x value given.

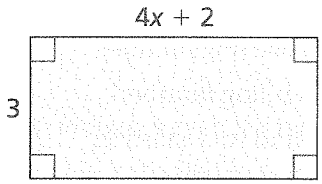
   a. 1 **b.** 2 **c.** 3 **d.** −3

   e. −11 **f.** $\frac{1}{2}$ **g.** 197

   h. Use the Distributive Property to simplify the expression.

5. The lengths and widths of four rectangles are given below. For each rectangle, find an expression for the area and an expression for the perimeter.

   a. length: $4x + 2$ width: 3



   b. length: 7 width: $x - 4$

   c. length: $6x - 8$ width: $\frac{1}{2}$

   d. length: $10 - 2x$ width: 9

On Your Own

6. Use the basic rules of arithmetic and what you know about like terms. Decide which expressions equal the expression $4x + 2y$. If an expression does not equal $4x + 2y$, explain why it does not.

   a. $4x + 6y + (-4y)$

   b. $4(x + y) - 2y$

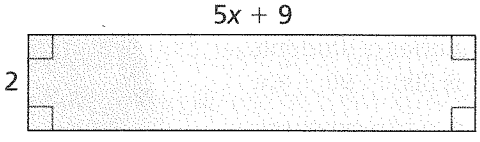
   c. $6xy$

   d. $(4x)(2y)$

   e. $x + x + x + x + x + x - y - y - y - y + 2(3y - x)$

7. The lengths and widths of four rectangles are given below. For each rectangle, find an expression for the area and an expression for the perimeter.

   a. length: $5x + 9$ width: 2



   b. length: 11 width: $x - 3$

   c. length: $2x + 9$ width: 2

   d. length: $8 - 3x$ width: 5

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8. Use the two number tricks.

Trick 1

- Choose a number.
- Add 6.
- Multiply by 3.
- Subtract 4.
- Multiply by 2.
- Add 2.
- Divide by 6.
- What is your ending number?

Trick 2

- Choose a number.
- Multiply by 2.
- Subtract 6.
- Multiply by 5.
- Add 50.
- Divide by 10.
- Subtract your starting number.
- What is your ending number?

a. Using one of these tricks, Spiro the Spectacular can find your starting number. Which trick can he use? How does he find your number?

b. Even Spiro the Spectacular cannot find your starting number using the other trick. Explain why not.

9. Evaluate the expression $2(3m + 5) - 5(m + 1) - 4$ for each value of m.

a. 3 **b.** 17 **c.** -2

d. 4 **e.** $\frac{1}{3}$ **f.** $-\frac{4}{11}$

g. Simplify the expression.

10. **Standardized Test Prep** Simplify the expression $7(6t + 2) + 3 - 5(t + 1)$.

A. 49 **B.** $37t + 12$

C. $42t + 17$ **D.** $47t + 22$

11. When Derman shops, he tries to buy all his clothes at the same store. Derman explains, "A store charges 6% sales tax on each purchase. Suppose you buy a shirt that costs s dollars at one store and a pair of pants that costs p dollars at another store. You pay $0.06 \cdot s$ in taxes for the shirt and $0.06 \cdot p$ in taxes for the pants. So you're paying taxes *twice*. If you buy them at the same store, you only pay the 6% once. So you're saving money."

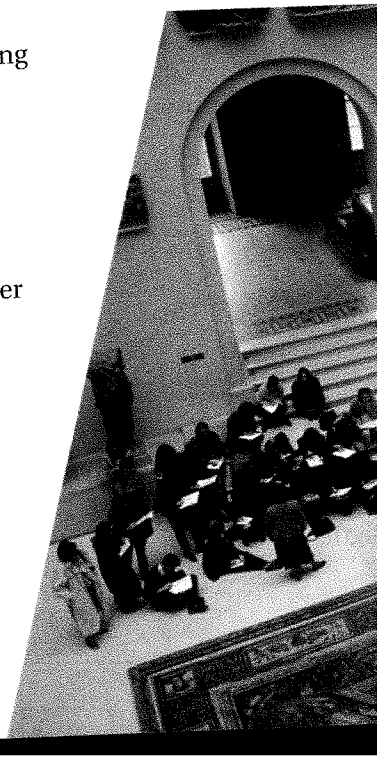
Is Derman correct? Using expressions, explain.

12. Jabari's art club raises \$300 to pay for a trip to a local museum. The bus costs \$90, and admission is \$6 per student. Jabari writes an expression for the amount the club has left based on the number of students who go to the museum. If x is the number of students who go to the museum, the amount remaining is $300 - 90 - 6x$.

a. What is the amount remaining after paying field trip expenses for 28 students? For 29 students? For 30 students?

b. What is the greatest number of students who can go on the field trip for \$300?

In Exercise 12, suppo
student admission pr
changed to \$8. How
this affect the expres
$300 - 90 - 6x$?



Maintain Your Skills

13. Evaluate the expression $x \cdot (x + 1)$ for each value of x.

a. 1 **b.** 2 **c.** 3 **d.** 4

e. 11 **f.** -3 **g.** -7

h. Explain why $x \cdot (x + 1)$ is always even if x is an integer.

14. Evaluate the expression $2 \cdot (x + 1)$ for each value of x.

a. 1 **b.** 2 **c.** 3 **d.** 4

e. 11 **f.** -3 **g.** -7

h. Explain why $2 \cdot (x + 1)$ is always even if x is an integer.

i. Explain why $2 \cdot x + 1$ is always odd if x is an integer.

15. Simplify each expression.

a. $x + 2x + 3x + 4x + 5x - 12x$

b. $x + 2x + 3x + 4x + 5x - 13x$

c. $x + 2x + 3x + 4x + 5x - 14x$

d. $x + 2x + 3x + 4x + 5x - 15x$

e. $x + 2x + 3x + 4x + 5x - 16x$

f. $x + 2x + 3x + 4x + 5x - 17x$

16. Evaluate the expression $\frac{1 - x^2}{1 - x}$ for each value of x.

a. 2 **b.** 3

c. -3 **d.** -11

e. $\frac{1}{2}$ **f.** 197

g. Identify a pattern in the results.

17. Evaluate the expression $(x + 1)^2 - x^2$ for each value of x.

a. 2

b. 3

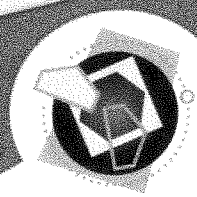
c. -3

d. -11

e. $\frac{1}{2}$

f. 197

g. Identify a pattern in the results.



Exercises Practicing Habits of Mind

Check Your Understanding

1. Express each sentence using variables.
   a. Dividing is the same as multiplying by the reciprocal.
   b. Subtracting is the same as adding the opposite.
   c. If you have a product of two numbers, and you find the products of the opposites of the numbers, you get the same result.
   d. If you multiply two numbers together and the result is 1, then the numbers are reciprocals.
2. The Zero-Product Property states that if the product of two numbers is zero, then one of the numbers equals zero. You can write the property using symbols this way.

   If $ab = 0$, then $a = 0$ or $b = 0$.

   The steps show you a proof of this property. Start with the equation $ab = 0$.
   a. If $a = 0$, then you can stop doing the proof. Explain.
   b. Assume $a \neq 0$. Then a has a reciprocal. Explain.
   c. Since a has a reciprocal $\left(\frac{1}{a}\right)$, you can use a basic rule and multiply both sides of the equation by this reciprocal. What effect does this have on the left side of the equation? On the right side of the equation?
   d. Explain why these steps prove the Zero-Product Property.

Remember...

Sometimes you will
the term *negative*
additive inverse ins
of *opposite*. They a
mean the same thi

On Your Own

3. The binary operation ♥ is defined by the following rule.

   $x \heartsuit y = 3x + y$

   a. Explain how to find $4 \heartsuit 6 = 18$.
   b. Evaluate $6 \heartsuit 4$.
   c. Is ♥ commutative? In other words, does ♥ have an any-order property?
4. The binary operation ♠ is defined by the rule $x \spadesuit y = -3(x + y)$.
   a. Is ♠ commutative? In other words, does ♠ have an any-order property? Explain.
   b. Is ♠ associative? In other words, does ♠ have an any-grouping property? Explain.

The prefix *bi-* me
"two." A **binary**
operation takes
numbers and pr
one number.

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5. Simplify.

   a. $2(x + 2) - (x + 2)$

   b. $2(x + 2) - (x + 2) + 4(x + 2) - 3(x + 2)$

   c. $2(x + 2) - (x + 2) + 4(x + 2) - 3(x + 2) + 6(x + 2) - 5(x + 2)$

   d. $2(x + 2) - (x + 2) + 4(x + 2) - 3(x + 2) + 6(x + 2) - 5(x + 2) + 8(x + 2) - 7(x + 2)$

   e. $2(x + 2) - (x + 2) + 4(x + 2) - 3(x + 2) + 6(x + 2) - 5(x + 2) + 8(x + 2) - 7(x + 2) + 10(x + 2) - 9(x + 2)$

   f. Evaluate each simplified expression for $x = -2$. What is the pattern in your results? Explain.

For Exercises 6 and 7, use the any-order, any-grouping properties and the Distributive Property to simplify each expression. Remember to combine like terms.

6. a. $4(x + 2) + 11$
   b. $x + 2(5 + 2x)$
   c. $9(2x - 5) - 3$
   d. $5(x - 1) + 8(x + 1)$
   e. $7(x + 1) + (7x + 7)$
   f. $7(x + 1) + (-1)(7x + 7)$

7. a. $2(x + 4) + 7$
   b. $13 + 3(1 + 2x)$
   c. $3(2x - 5) - 8$
   d. $4(x + 3) + 7(x + 3)$
   e. $6(3 - 2x) - 3(x + 1)$
   f. $4(x - 7) - 2(2 - 3x)$

8. Here is one of Maya the Magnificent's number tricks.

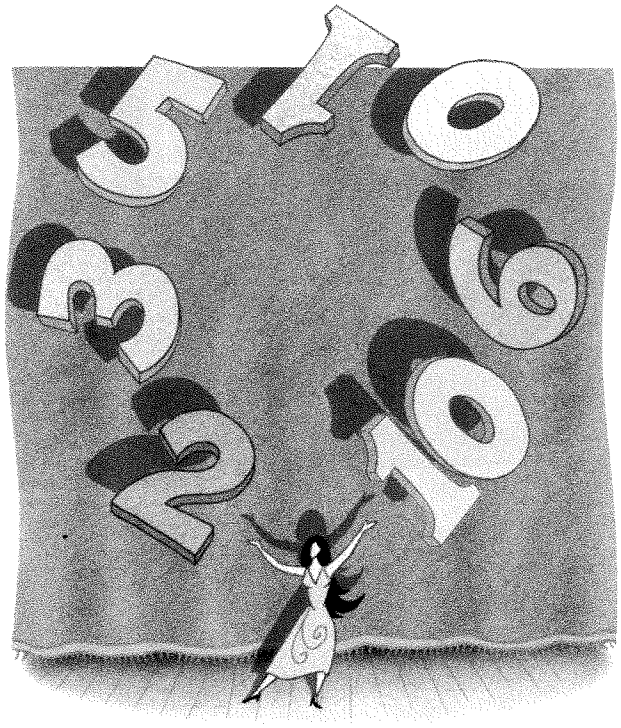
   - Choose a number.
   - Add 6.
   - Multiply by 3.
   - Subtract 10.
   - Multiply by 2.
   - Add 50.
   - Divide by 6.

   Maya says, "I take the ending number and subtract 11. That's your starting number."

   a. Let the starting number equal x. Write the result after each step. Simplify each expression after each step.

   b. Identify four places where you used a basic rule to simplify an expression.

   c. Explain why Maya only needs to subtract 11 to get the starting number.



9. Standardized Test Prep Define the binary operation $\otimes$ with the rule

$$a \otimes b = ab + a.$$

Which of the following statements is true?

A. The binary operation $\otimes$ is associative but not commutative.

B. The binary operation $\otimes$ is commutative but not associative.

C. The binary operation $\otimes$ is both associative and commutative.

D. The binary operation $\otimes$ is neither associative nor commutative.

10. Take It Further Tony buys CDs at a music store Web site. He says, "You get a great deal. You get 20% off the price of a CD. You still have to add 5% sales tax. But you find the sales tax after the discount. So you save money on taxes, too."

How much money does Tony save by getting the discount *before* adding the sales tax? Explain by using expressions.

Start by writing an expression for the co of a CD in which you calculate the sales ta after the discount.

Maintain Your Skills

Find each product. Look for patterns.

11. a. $(5 \times 4) \times 2$

b. $(5 \times 213) \times 2$

c. $(5 \times 91{,}827) \times 2$

d. Describe a pattern.

12. a. $(25 \times 17) \times 4$

b. $(25 \times 22) \times 4$

c. $(25 \times 197) \times 4$

d. Describe the pattern.

13. a. $\left(\frac{10}{13} \times 54\right) \times 13$

b. $\left(\frac{10}{13} \times 81\right) \times 13$

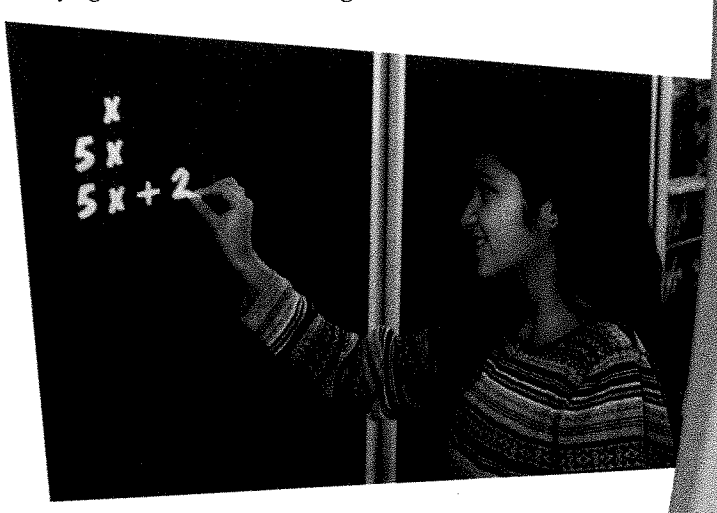
c. $\left(\frac{10}{13} \times 1113\right) \times 13$

d. Describe a pattern.

Mathematical Reflections 2A

In this investigation, you used and invented number tricks. You learned to write a variable expression to model a situation. Using the basic rules of arithmetic, you evaluated and simplified expressions. These questions will help you summarize what you have learned.

1. To build a rectangular dog pen, Cheng uses a wall of his house for one of the long sides. Let ℓ equal the length of the longer side. Let w equal the length of the shorter side.
   - **a.** Write an expression for the amount of fencing Cheng needs to build the pen.
   - **b.** How much fencing does Cheng need if he wants a width of 8 feet and a length of 12 feet?
   - **c.** How much fencing does Cheng need if he wants a width of 5 feet and a length of 20 feet?
   - **d.** Suppose the length is 9 feet more than the width. Use only *one* variable to write an expression for the amount of fencing Cheng needs.
2. Evaluate the expression $4(2x + 3) + 2(x + 1) - 7$ for each value of x.
   - **a.** 1 **b.** 6 **c.** -2 **d.** $\frac{1}{2}$
   - **e.** Simplify the expression. When you evaluate the simplified expression for $x = 1$ and $x = 6$, do you get the same results as in part (a) and part (b)? Do you get the same result for every value of x?
3. Why are variables useful?
4. How can you invent a number trick that always gives the same ending number?
5. Use the following steps.
   - Choose a number.
   - Multiply by 5.
   - Add 2.
   - Multiply by 3.
   - Subtract 8.
   - **a.** Choose the number 4. What ending number do you get?
   - **b.** Choose the variable x. What ending expression do you get?



Puedes comprender x, $5x$, $5x + 2$, . . . , sin importar la lengua que hables.

Vocabulary

In this investigation, you saw these terms. Make sure you understand what each one means and how to use it.

- **binary operation**
- **combining like terms**
- **evaluate**
- **expression**
- **like terms**
- **variable**